

Sri Chaitanya's Solutions to

IIT – JEE - 2011

(PAPER – 2)

Time: 3 Hours

Maximum Marks: 237

Please read the instructions carefully. You are allotted 5 minutes specifically for this purpose

INSTRUCTIONS

A. General :

1. The **question paper CODE** is printed on the right hand top corner of this sheet and also on the back page (page no 32 of this booklet)
2. No additional sheets will be provided for rough work
3. Blank papers, clipboards, log tables, slide rules, calculators, cellular phones, pagers and electronic gadgets in any form are not allowed
4. Write your name and registration number in the space provided on the back page of this booklet
5. The answer sheet, a machine - gradable Objective Response Sheet (ORS), is provided separately.
6. **DO NOT TAMPER WITH/MULTILATE THE ORS OR THE BOOKLET.**
7. Do not break the seals of the question paper booklet before instructed to do so by the invigilators.
8. The question paper contains 32 pages having 60 questions
9. On breaking the seals, please check that all the questions are legible.

B. Filling the bottom half of the ORS :

10. The ORS has **CODE** printed on its lower and upper Parts.
11. Make sure the CODE on the ORS is the same as that on this booklet. **If the Codes do not match, ask for a change of the Booklet.**
12. Write your Registration No., Name and Name of centre and sign with pen in appropriate boxes. **Do not write these anywhere else.** Darken the appropriate bubbles below your registration number with **HB pencil**.

C. Question paper format and marking scheme :

13. The question paper consists of **3 Parts** (Chemistry, Mathematics and Physics), and each part consists of **four** Sections
14. In **Section I** (Total Marks : 24), for each question you will be awarded **3 marks** if you darken **ONLY** the bubble corresponding to the correct answer and **zero marks** if no bubble is darkened. In all other cases, **minus one (-1)** mark will be awarded.
15. In **Section II** (Total Marks : 16) for each question you will be awarded **4 marks** if you darken **ALL** the bubble(s) corresponding to the correct answer(s) **ONLY** and **zero marks** otherwise. There are **no negative** marks in this section.
16. In **Section III** (Total Marks : 24) for each question you will be awarded **4 marks** if you darken **ONLY** the bubble corresponding to the correct answer and **zero marks** otherwise. There are **no negative** marks in this section.
17. In **Section IV** (Total Marks : 16), for each question you will be awarded **2 marks** for each row in which you have darkened **ALL** the bubble(s) corresponding to the correct answer(s) **ONLY** and **zero marks** otherwise. Thus each question in this section carries a **maximum of 8 marks**. There are **no negative marks** in this section.

IIT JEE 2011 (PAPER - II)

PART - I CHEMISTRY

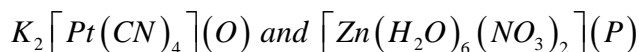
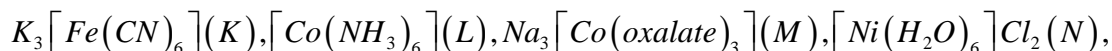
CHEMISTRY

SECTION - I (Total Marks : 24)

(SINGLE CORRECT CHOICE TYPE)

This section contains **8 multiple choice questions**. Each question has four choices A,B,C and D out of which **ONLY ONE** is correct.

1. Among the following complexes ($K - P$),



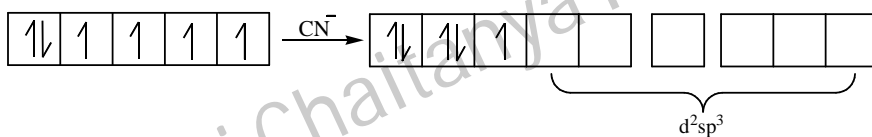
the diamagnetic complexes are

- a) K,L,M,N b) K, M, O, P c) L, M, O,P d) L, M, N, O

Ans : C

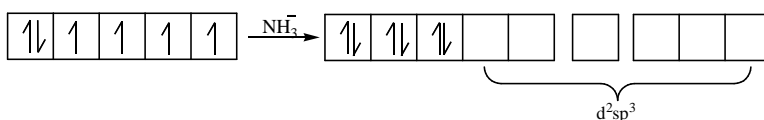
Sol : (K) $K_3[Fe(CN)_6]$ - paramagnetic

d^2sp^3



(L) $[Co(NH_3)_6]Cl_3$ - diamagnetic

$Co^{+3} = d^6$



(M) $[Co(oxalate)_3]$ - same as above

$Co^{+3} = d^6$ - due to chelation d^2SP^3

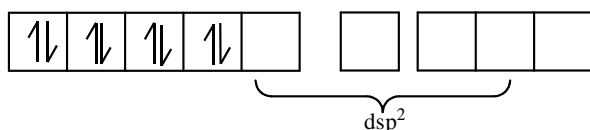
hybridisation takes place

Hence dia magnetic

(N) $[Ni(H_2O)_6]Cl_2$

$Ni^{+2} = d^8 \xrightarrow{H_2O} \text{Paramagnetic}$

(P) $K_2[Pt(CN)_4] = dsp^2$



$Zn^{+2} = d^{10}$ - dia Magnetic

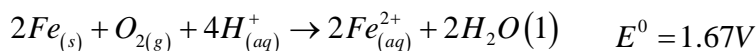
2. Passing H_2S gas into a mixture of Mn^{2+} , Ni^{2+} , Cu^{2+} and Hg^{2+} ions in an acidified aqueous solution precipitates

- a) CuS and HgS b) MnS and CuS c) MnS and NiS d) NiS and HgS

Ans : A

Sol : H_2S in acidic Medium is group reagent for 2nd group cation Hence only CuS , HgS are precipitated

3. Consider the following cell reaction :



At $[Fe^{2+}] = 10^{-3} M$, $P(O_2) = 0.1$ atm and $pH = 3$, the cell potential at $25^\circ C$ is

- a) 1.47 V b) 1.77 V c) 1.84 V d) 1.57 V

Ans : D

$$\text{Sol: } E_{\text{Cell}} = E_{\text{Cell}}^0 - \frac{0.0591}{4} \text{Log} \left(\frac{[Fe^{2+}]^2}{P_{O_2} \cdot [H^+]^4} \right)$$

$$= 1.67 - \frac{0.0591}{4} \text{Log} \left(\frac{[10^{-3}]^2}{(0.1) \cdot [10^{-3}]^4} \right)$$

$$= 1.57$$

4. The freezing point (in $^\circ C$) of a solution containing 0.1 g of $K_3[Fe(CN)_6]$ (Mol.Wt.329) in 100 g of water ($K_f = 1.86 K \text{ kg mol}^{-1}$) is

- a) -2.3×10^{-2} b) -5.7×10^{-2} c) -5.7×10^{-3} d) -1.2×10^{-2}

Ans : A

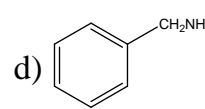
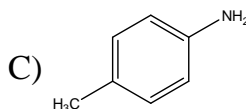
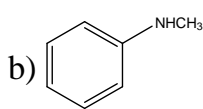
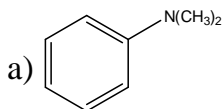
Sol: $\Delta T_f = iK_f m$, where $i = 4$

$$= 0.0226$$

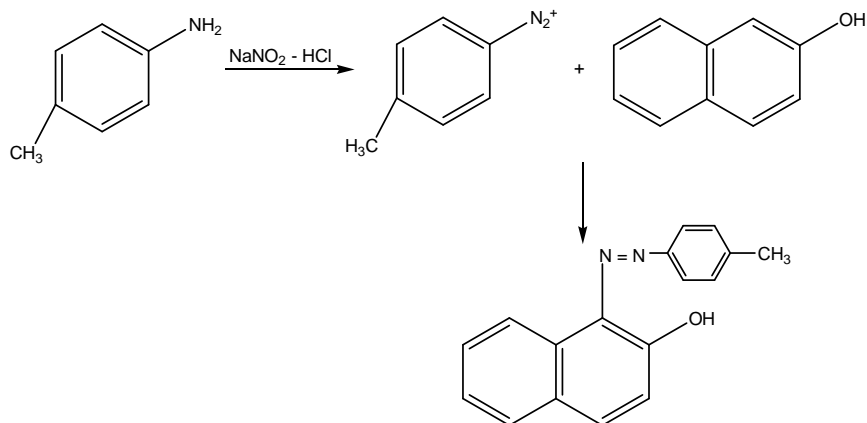
$$T_f = -2.261 \times 10^{-2}$$

$$= -2.3 \times 10^{-2}$$

5. Amongst the compounds given, the one that would form a brilliant colored dye on treatment with $NaNO_2$ in dil. HCl followed by addition to an alkaline solution of β -naphthol is

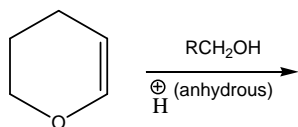


Ans : C



Sol :

6. The major product of the following reaction is



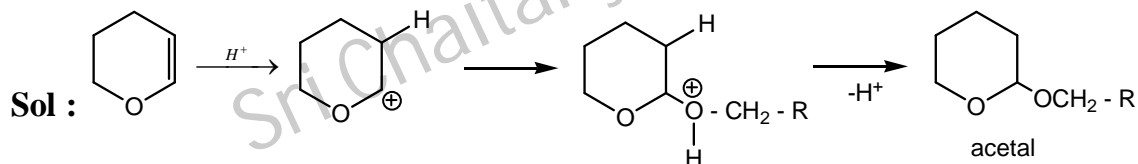
a) c hemiacetal

b) an acetal

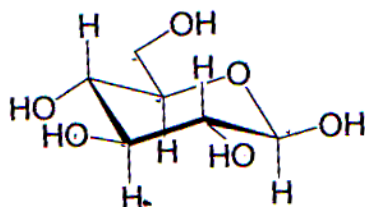
c) an ether

d) an ester

Ans : B



7. The following carbohydrate is



a) a ketohexose

b) an aldohexose

c) an α -furanosed) an α -pyranose

Ans : B

Sol :



8. Oxidation states of the metal in the minerals haematite and magnetic, respectively, are
- a) II, III in haematite and II in magnetic b) II, III in haematite and II in magnetic
c) II in haematite and II, II in magnetic d) III in haematite and II, III in magnetic

Ans : D

Sol : Haematite : Fe_2O_3 (+3)

Magnetite : $\text{Fe}_3\text{O}_4 \rightarrow \text{Fe}_2\text{O}_3(+3)+\text{FeO} (+2)$
(Mixed oxide)

SECTION – II (Total Marks : 16)

(MULTIPLE CORRECT ANSWER TYPE)

This section contains **4 multiple choice questions**. Each question has four choices A,B,C and D out of which **ONE or MORE** may be correct.

9. The equilibrium

in aqueous medium at 25°C shifts towards the left in the presence of



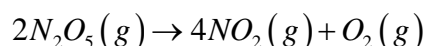
- a) NO_3^- b) Cl^- c) SCN^- d) CN^-

Ans : BCD

Sol : Because in Aq. Medium Cu^{+1} is Highly unstable and undergoes disproportionation.

Cu^+ is stable only with insoluble Cu_2Cl_2 , $\text{Cu}_2(\text{CN})_2$, $\text{Cu}_2(\text{SCN})_2$ in aq. medium hence with these ligands. Equilibrium moves to left side

10. For the first order reaction



- a) The concentration of the reactant decreases exponentially with time
b) The half-life of the reaction decreases with increasing temperature
c) The half-life of the reaction depends on the initial concentration of the reactant
d) The reaction proceeds to 99.6% completion in eight half-life duration.

Ans : ABD

Sol: (a) $C = C_0 \cdot e^{-kt}$

$$(b) t_{1/2} = \frac{0.693}{k}$$

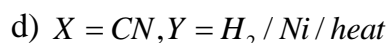
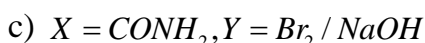
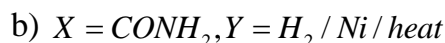
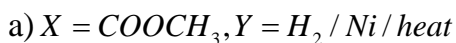
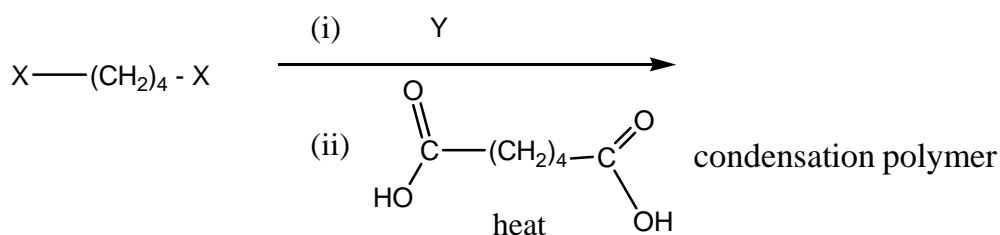
Since k increases with increasing of temperature

$\therefore t_{1/2}$ decreases

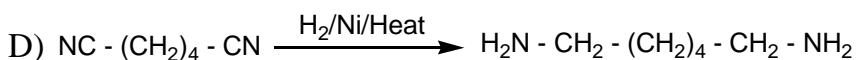
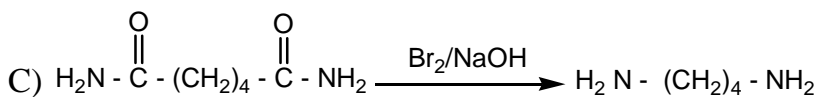
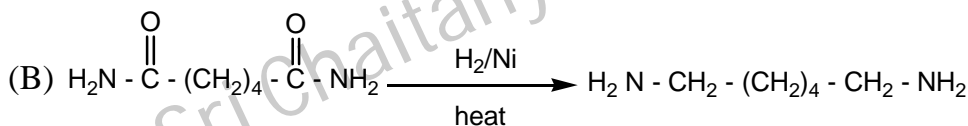
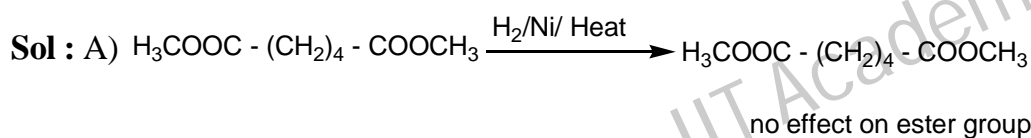
$$(c) t_{1/2} \propto a^0$$

$$(d) t_{99.6} = 8t_{1/2}$$

11. The correct functional group X and the reagent/reaction conditions Y in the following scheme are



Ans: BCD



12. Reduction of the metal centre in aqueous permanganate ion involves

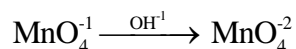
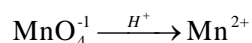
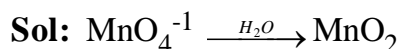
a) 3 electrons in neutral medium

b) 5 electrons in neutral medium

c) 3 electrons in alkaline medium

d) 5 electrons in acidic medium

Ans: A,D



SECTION – III (Total Marks : 24)**(INTEGER ANSWER TYPE)**

This section contains **6 questions**. The answer to each of the questions is a **single-digit integer**, ranging from 0 to 9. The bubble corresponding to the correct answer is to be darkened in the ORS.

13. The volume (in mL) of 0.1 M AgNO_3 required for complete precipitation of chloride ions present in 30 mL of 0.01 M solution of $[\text{Cr}(\text{H}_2\text{O})_5\text{Cl}]\text{Cl}_2$, as silver chloride is close to

Ans : 6

Sol: m. moles of $\text{AgNO}_3 = 2 \times$ m.moles of $[\text{Cr}(\text{H}_2\text{O})_5\text{Cl}]\text{Cl}_2$

$$0.1 \times V_{\text{ml}} = 2 \times 0.3$$

$$V_{\text{ml}} = 6$$

14. In 1 L saturated solution of AgCl [$K_{\text{sp}}(\text{AgCl}) = 1.6 \times 10^{-10}$], 0.1 mol of CuCl [$K_{\text{sp}}(\text{CuCl}) = 1.0 \times 10^{-6}$] is added. The resultant concentration of Ag^+ in the solution is 1.6×10^{-x} . The value of “x” is

Ans: 7

Sol: $K_{\text{sp}}(\text{AgCl}) = [\text{Ag}^+][\text{Cl}^-]$

$$[\text{Cl}^-] = 10^{-3} \text{ from } K_{\text{sp}}(\text{CuCl})$$

$$1.6 \times 10^{-10} = [\text{Ag}^+] 10^{-3}$$

$$[\text{Ag}^+] = 1.6 \times 10^{-7}$$

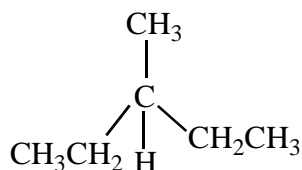
$$\therefore x = 7$$

15. The number of hexagonal faces that are present in a truncated octahedron is

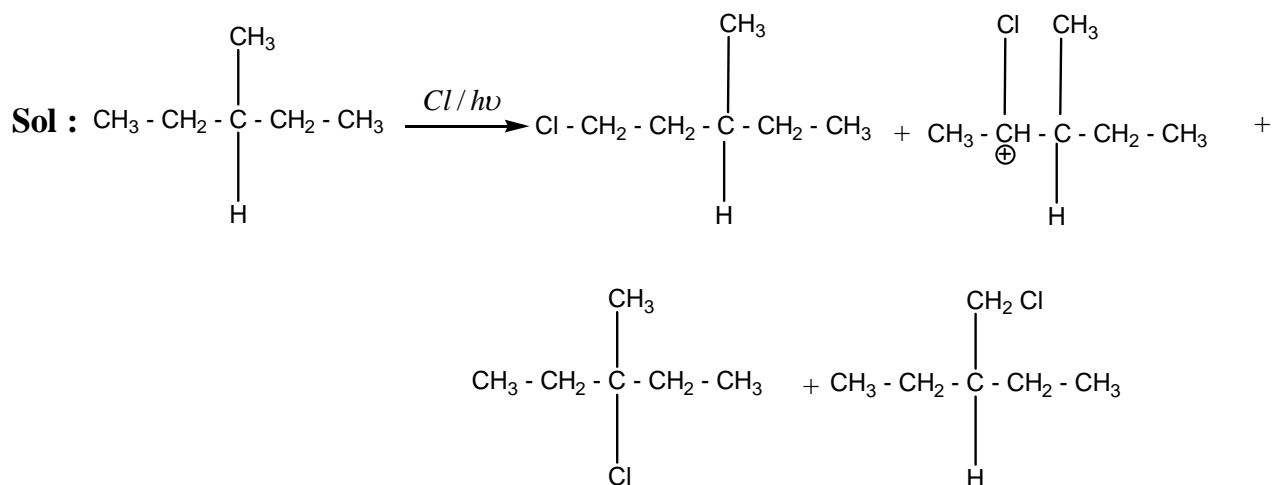
Ans : 8

Sol : Truncated octahedron. It is an archimedean solid. It has 14 faces (8 regular hexagonal and 6 square)

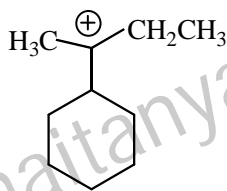
16. The maximum number of isomers (including stereoisomers) that are possible on monochlorination of the following compound, is



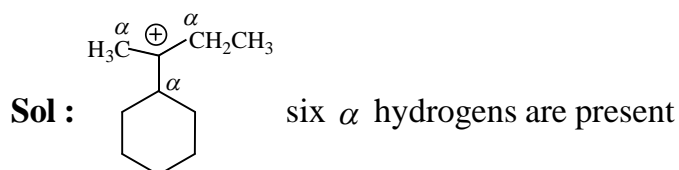
Ans : 8



17. The total number of contributing structures showing hyperconjugation (involving C-H bonds) for the following carbocation is

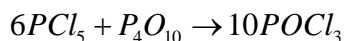
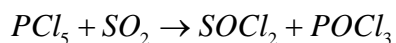
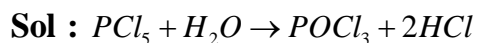


Ans : 6



18. Among the following, the number of compounds than can react with PCl_5 to give POCl_3 is $\text{O}_2, \text{CO}_2, \text{SO}_2, \text{H}_2\text{O}, \text{H}_2\text{SO}_4, \text{P}_4\text{O}_{10}$

Ans : 4



SECTION – IV(Total Marks : 16)

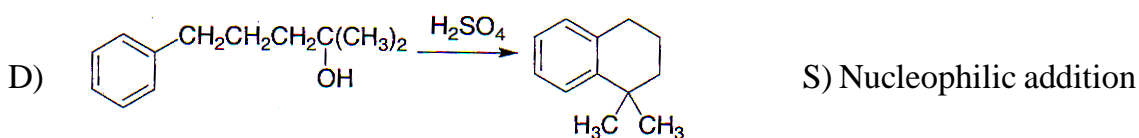
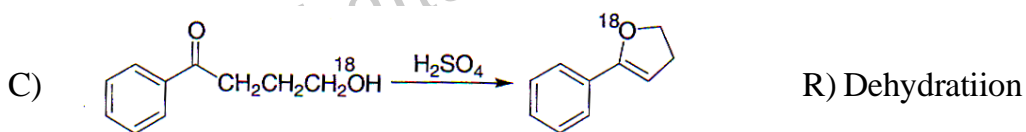
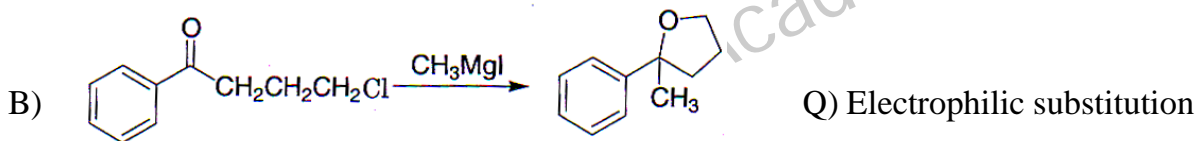
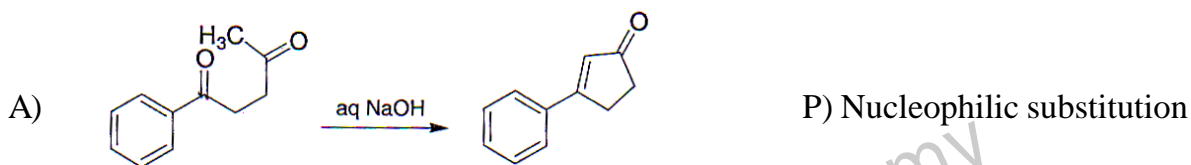
(MATRIX-MATCH TYPE)

This section contains **2 questions**. Each question has **four statements** A,B,C and D given in **Column I** and **five statements** p,q,r,s and t in **Column II**. Any given statement in Column I can have correct matching with **ONE** or **MORE** statement(s) given in Column II. For example, if for a given question, statement B matches with the statement given in q and r, then for the particular question, against statement B, darken the bubbles corresponding to q and r in the ORS.

19. Match the reaction in **column I** with appropriate types of steps/reactive intermediate involved in these reactions as given in **column II**

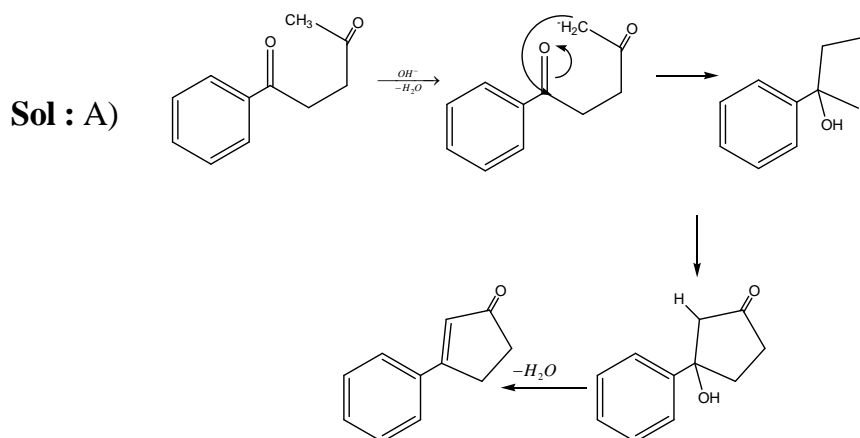
Column - I

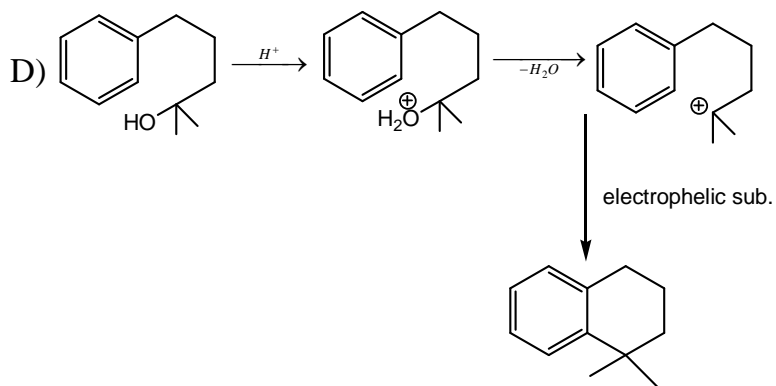
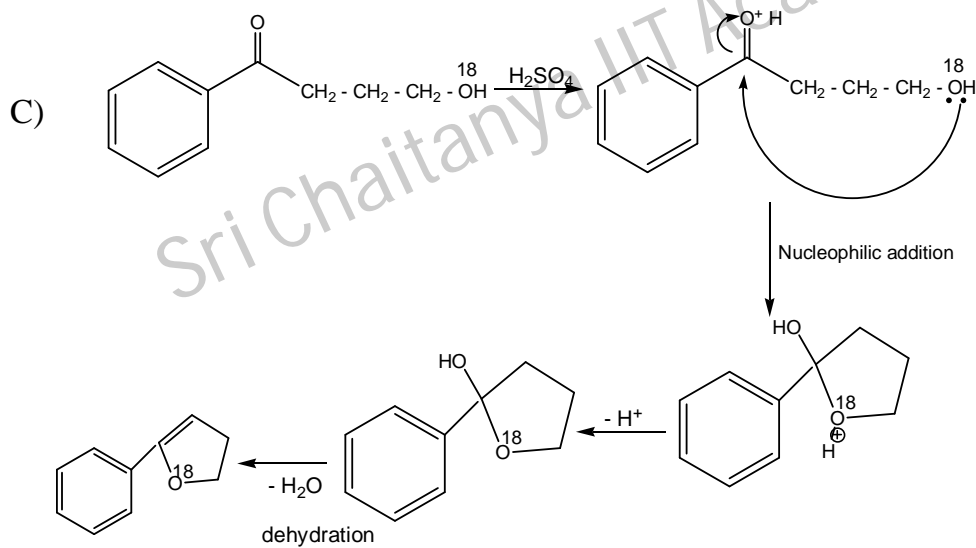
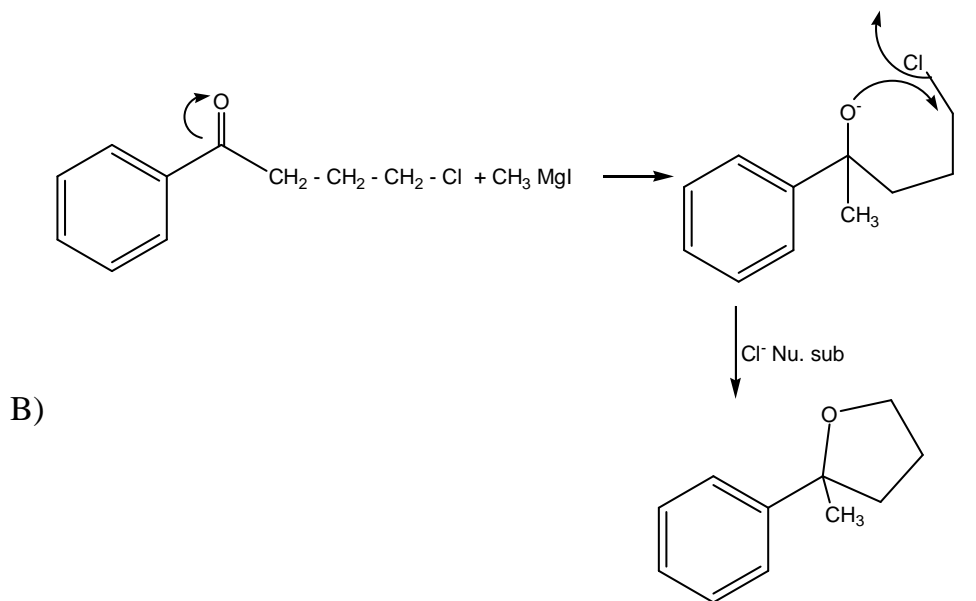
Column -II



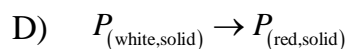
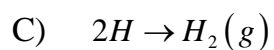
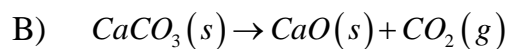
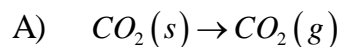
T) Carbanion

Ans : A → rst; B → pst; C → rs; D → qr





20. Match the transformations in **column I** with appropriate options in **column II**

Column I**Column II**

(P) phase transition

(Q) allotropic change

(R) ΔH is positive(S) ΔS is positive(T) ΔS is negative

Ans: A \rightarrow p,r,s; B \rightarrow r,s; C \rightarrow t; D \rightarrow q,t

Sol : Conceptual

PART II : PHYSICS

SECTION – I (Total Marks : 24)

(SINGLE CORRECT CHOICE TYPE)

This section contains **8 multiple choice questions**. Each question has four choices (A), (B), (C) and (D) for its answer, out of which **ONLY ONE** is correct

21. A satellite is moving with a constant speed 'V' in a circular orbit about the earth. An object of mass 'm' is ejected from the satellite such that it just escapes from the gravitation pull of the earth. At the time of its ejection, the kinetic energy of the object is

- a) $\frac{1}{2}mV^2$ b) mV^2 c) $\frac{3}{2}mV^2$ d) $2mV^2$

Ans : B

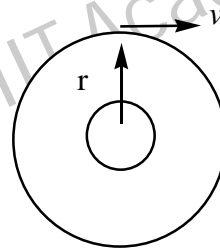
$$\text{Sol : } KE_i = \frac{1}{2}mV^2 = \frac{|P.E.}_i}{2}$$

$$PE_i = -\frac{Gm}{r}$$

$$\text{finally } KE + PE_r = 0$$

$$KE = -PE_r$$

$$= +\frac{Gm}{r} = mV^2$$



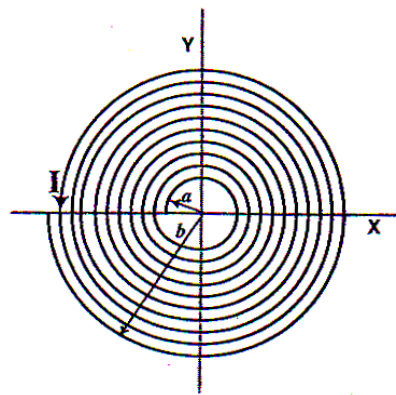
22. A long insulated copper wire is closely wound as a spiral of 'N' turns. The spiral has inner radius ' α ' and outer radius 'b'. The spiral lies in the X - Y plane and a steady current 'I' flows through the wire. The Z - component of the magnetic field at the center of the spiral is

a) $\frac{\mu_0 NI}{2(b-a)} \ln\left(\frac{b}{a}\right)$

b) $\frac{\mu_0 NI}{2(b-a)} \ln\left(\frac{b+a}{b-a}\right)$

c) $\frac{\mu_0 NI}{2b} \ln\left(\frac{b}{a}\right)$

d) $\frac{\mu_0 NI}{2b} \ln\left(\frac{b+a}{b-a}\right)$

**Ans : A**

Sol : $\frac{N}{b-a} = \frac{dN}{dr} \dots(1)$

$$dB_c = \frac{\mu_0}{2} \frac{(dNI)}{r}$$

$$dB_c = \frac{\mu_0 I}{2} \frac{1}{r} dr \left(\frac{N}{b-a} \right)$$

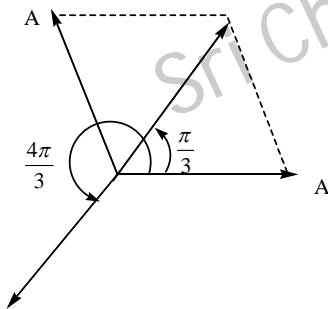
$$B_c = \frac{\mu_0 IN}{2(b-a)} \ln \frac{b}{a}$$

23. A point mass is subjected to two simultaneous sinusoidal displacements in x - direction, $x_1(t) = A \sin \omega t$ and $x_2(t) = A \sin \left(\omega t + \frac{2\pi}{3} \right)$. Adding a third sinusoidal displacement $x_3(t) = B \sin(\omega t + \phi)$ brings the mass to a complete rest. The values of B and ϕ are

- a) $\sqrt{2}A, \frac{3\pi}{4}$ b) $A, \frac{4\pi}{3}$ c) $\sqrt{3}A, \frac{5\pi}{6}$ d) $A, \frac{\pi}{3}$

Ans : B

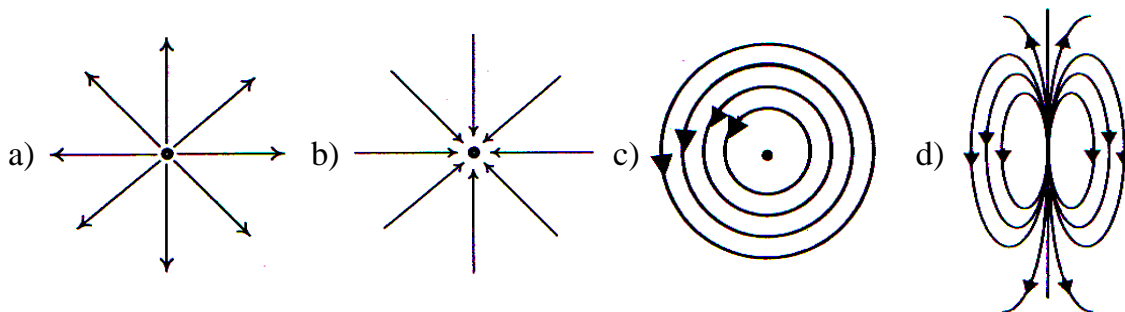
Sol :



$$A_2 = A; \pi/3$$

$$B = A; \text{ at } \phi = \frac{4\pi}{3}$$

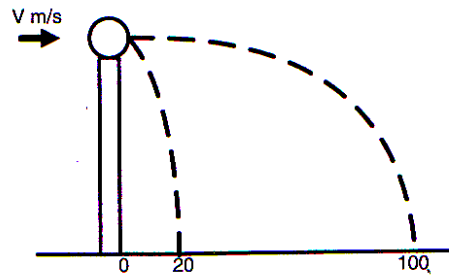
24. Which of the field patterns given below is valid for electric field as well as for magnetic field ?



Ans : C

Sol : Induced electric field may be a closed loop

25. A ball of mass 0.2 kg rests on a vertical of heights 5 m. A bullet of mass 0.01 kg, travelling with a velocity V m/s in a horizontal direction, hits the centre of the ball. After the collision, the ball and bullet travel independently. The ball hits the ground at a distance of 20 m and the bullet at a distance of 100 m from the foot of the post. The initial velocity V of the bullet is



- a) 250 m/s b) $250\sqrt{2}$ m/s c) 400 m/s d) 500 m/s

Ans : D

Sol : time of fall = t

$$\frac{1}{2}gt^2 = 5$$

$$t = 1 \text{ sec}$$

$$V_{ball} \times 1 = 20; V_{ball} = 20 \text{ m/s}$$

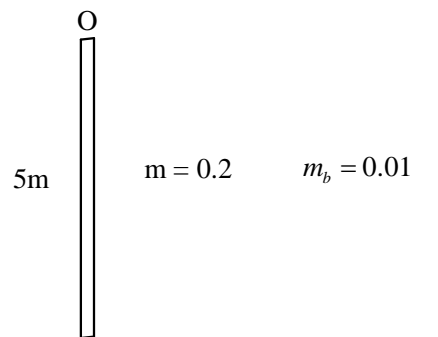
$$V_{bullet} \times 1 = 100; V_{bullet} = 100 \text{ m/s}$$

$$m_{bullet} V = m_{bullet} V_{bullet} + m_{ball} \cdot V_{ball}$$

$$0.01v = 0.01 \times 100 + 0.2 \times 20$$

$$0.01v = 1 + 4 = 5$$

$$v = 500 \text{ m/s}$$



26. The density of a solid ball is to be determined in an experiment. The diameter of the ball is measured with a screw gauge, whose pitch is 0.5 mm and there are 50 divisions on the circular scale. The reading on the main scale is 2.5 mm and that on the circular scale is 20 divisions. If the measured mass of the ball has a relative error of 2%, the relative percentage error in the density is

- a) 0.9% b) 2.4% c) 3.1% d) 4.2%

Ans : C

Sol : 1 msd = 0.5mm

csd has 50 division

$$50\text{csd} = 1\text{msd}$$

$$50 \text{ csd} = 0.5 \text{ mm} \quad \dots\dots(1)$$

$$\text{least count} = 0.01\text{mm}$$

diameter = d ;

$$d = 2.5 \text{ mm} + 20 \left(\frac{0.5}{50} \right) \text{ mm}$$

$$d = 2.7 \text{ mm}$$

$$m = \frac{4}{3} \pi \left(\frac{d^3}{8} \right) \cdot \rho$$

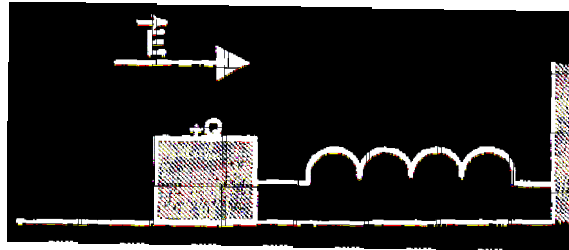
$$\left. \frac{d\rho}{\rho} \right|_{\max} = \frac{dm}{m} + 3 \frac{d(d)}{d}$$

$$\text{now } \frac{3d(d)}{d} = 3 \frac{(0.01)}{2.7} = \frac{1}{90}$$

$$\% \text{ value} \approx 1.1$$

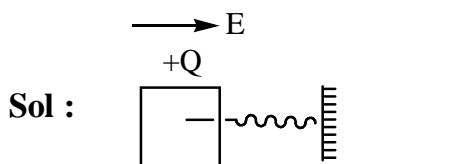
$$\left. \frac{d\rho}{\rho} 100 \right|_{\max} = 2 + 1.1 = 3.1$$

27. A wooden block performs SHM on a frictionless surface with frequency, ν_0 . The block carries a charge $+Q$ on its surface. If now a uniform electric field \vec{E} is switched - on as shown, then the SHM of the block will be



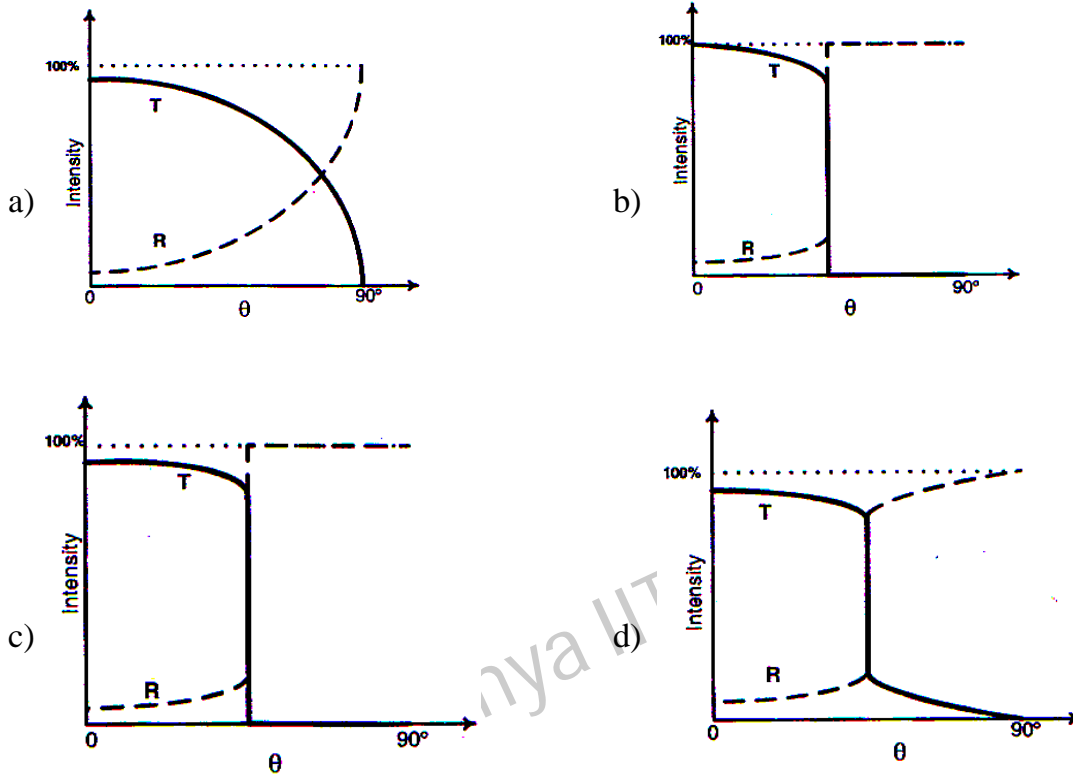
- of the same frequency and with shifted mean position
- of the same frequency and with the same mean position
- of changed frequency and with shifted mean position
- of changed frequency and with the same mean position

Ans : A



due to new constant force mean position is shifted but frequency of oscillation will not change

28. A light ray traveling in glass medium is incident on glass - air interface at an angle of incidence θ . The reflected (R) and transmitted (T) intensities, both as function of θ , are plotted. The correct sketch is



Ans : C

Sol : 100% reflection or transmission not possible

SECTION – II (Total Marks : 16)

(MULTIPLE CORRECT ANSWER(S) TYPE)

This section contains **4 multiple choice questions**. Each question has four choices (A), (B), (C) and (D) for its answer, out of which **ONE or MORE** may be correct

29. A series R-C circuit is connected to AC voltage source. Consider two cases ; (A) when C is without a dielectric medium and (B) when C is filled with dielectric of constant 4. The current I_R through the resistor and voltage V_C across the capacitor are compared in the two cases. Which of the following is / are true ?

a) $I_R^A > I_R^B$

b) $I_R^A < I_R^B$

c) $V_C^A > V_C^B$

d) $V_C^A < V_C^B$

Ans : BC

Sol : $z = \sqrt{R^2 + \frac{1}{\omega C}}$

$C_A < C_B \Rightarrow Z_A > Z_B$

$$\therefore I_R^A < I_R^B \Rightarrow (B) \text{ is correct}$$

$$V_C = X_C I = \frac{1}{\sqrt{wc}} \cdot \frac{\varepsilon}{\sqrt{R^2 + \frac{1}{wc}}} = \frac{\varepsilon}{\sqrt{R^2 wc + 1}}$$

$$C_A < C_B \Rightarrow V_C^A > V_C^B$$

$\Rightarrow (c)$ is correct

30. Which of the following statement(s) is/are correct ?

- a) If the electric field due to a point charge varies as $r^{-2.5}$ instead of r^{-2} , then the Gauss law will still be valid.
- b) The gauss law can be used to calculate the field distribution around an electric dipole.
- c) If the electric field between two point charges is zero somewhere, then the sign of the two charges is the same.
- d) The work done by the external force in moving a unit positive charge from point A at potential V_A to potential V_B is $(V_B - V_A)$

Ans : CD

Sol : Surface area $\propto r^2$

So., if and only if $E \propto r^{-2}$, $\oint \vec{E} \cdot d\vec{s}$ will be independent of r

$\therefore (A)$ is false

Around a dipole, $\oint \vec{E} \cdot d\vec{s} = q_{in}$, net = zero

So, for any type of Gaussian surface, RHS = 0

The equation will not be useful to calculate E .

$\Rightarrow (B)$ is false

$$\vec{E}_{net} = k \frac{q_1}{r_1^2} \cdot \hat{r}_1 + \frac{kq_2}{r_2^2} \cdot \hat{r}_2 = 0$$

Therefore, $\hat{r}_A = -\hat{r}_B$ (in between the charges)

$\therefore q_A$ & q_B must have same sign (c) is correct

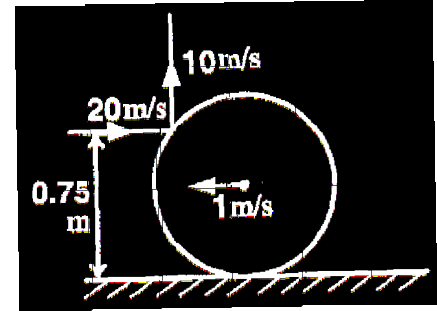
From the definition of work done by electric field,

$$W = q(V_B - V_A)$$

(D) is correct

31. A thin ring of mass 2 kg and radius 0.5 m is rolling without slipping on a horizontal plane with velocity 1 m/s. A small ball of mass 0.1 kg, moving velocity 20 m/s in the opposite direction, hits ring at a height of 0.75 m and goes vertically up with velocity 10 m/s. Immediately after the collision.

- the ring has pure rotation about its stationary CM.
- the ring comes to a complete stop
- friction between the ring and the ground is to the left.
- there is no friction between the ring and the ground.



Ans : A,C

Sol : $F_x dt = 0.1(0.20) = -2$

$$F_y dt = 0.1(10 - 0) = 1$$

for the ring, $N = Mg + F_y \approx F_y$

$$F_x dt = M(V_2 - V_1)$$

$$-2 = 2(V_2 - 1) \Rightarrow V_2 = 0$$

$$\tau dt : F_x dt \times 0.25 - F_y dt \times 0.25\sqrt{3}$$

$$2 \times 0.25 - 1 \times 0.25\sqrt{3}$$

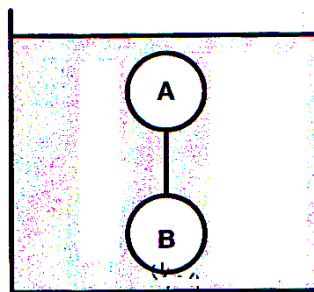
$$0.5 - 0.25\sqrt{3} = 0.067$$

$$L_i = MR^2 \omega = MVR = 2 \times 1 \times 0.5 = 1$$

$$L_f = L_i = \tau dt = 1 - 0.067 \neq 0$$

$$\therefore W_f \neq 0$$

32. Two solid spheres A and B of equal volumes but of different densities d_A and d_B are connected by a string. They are fully immersed in a fluid of density d_F . They get arranged into an equilibrium state as shown in the figure with a tension in the string. The arrangement is possible only if



a) $d_A - d_F$

b) $d_B > d_F$

c) $d_A > d_F$

d) $d_A + d_B = 2d_F$

Ans : ABD**Sol :** Equilibrium : $B_1 + B_2 = W_1 + W_2$

$$d_F(V + V)g = Vg(d_A + d_B)$$

$$2d_F = (d_A + d_B) \rightarrow (1)$$

If $d_B < d_F$, the string becomes black

∴ To keep the string tight,

$$d_A < d_F \quad \text{and} \quad d_B > d_F$$

∴ Ans : **ABD****SECTION -III (Total Marks : 24)****(INTEGER ANSWER TYPE)**

This section contains **6 questions**. The answer to each of the questions is a **single-digit integer**, ranging from 0 to 9. The bubble corresponding to the correct answer is to be darkened in the ORS.

33. A block of mass 0.18 kg is attached to a spring of force-constant 2N/m. The coefficient of friction between the block and the floor is 0.1. Initially the block is at rest and the spring is un-stretched. An impulse is given to the block as shown in the figure. The block slides a distance of 0.06m and comes to rest for the first time. The initial velocity of the block in m/s is $V = N/10$. Then N is

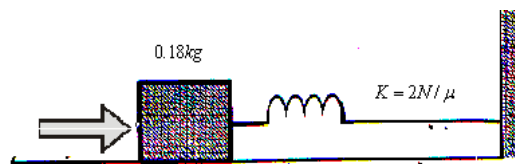
**Ans : 4**

$$\text{Sol : } \frac{1}{2}mv^2 = \mu mgx + \frac{1}{2}kx^2$$

$$\frac{1}{2}(0.18)V^2 = (0.1)(0.18)(10)(0.06) + \frac{1}{2}2(36 \times 10^{-4})$$

$$0.09 V^2 = 108 \times 10^{-4} + 36 \times 10^{-4}$$

$$0.09 V^2 = 10^{-4} [144]$$

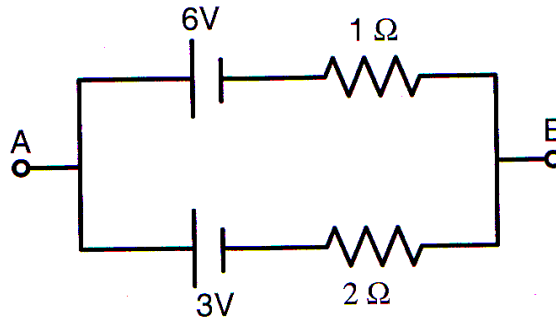


$$0.3 V = 10^{-2} (12)$$

$$V = 0.4 \text{ ms}^{-1} = \frac{N}{10}$$

$$\Rightarrow \boxed{N = 4}$$

34. Two batteries of different emfs and different internal resistance are connected as shown. The voltage across AB in volts is

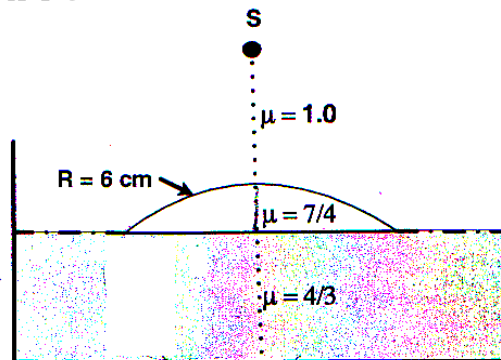


Ans : 5

$$\text{Sol : } V_{AB} = \frac{\varepsilon_1 r_2 + \varepsilon_2 r_1}{r_1 + r_2}$$

$$= \frac{6(2) + 3(1)}{1 + 2} = \boxed{5}$$

35. Water (with refraction index = $\frac{4}{3}$) in a tank is 18 cm deep. Oil of refractive index $\frac{7}{4}$ lies on water making a convex surface of radius of curvature 'R = 6 cm' as shown. Consider oil to act as a thin lens. An object 'S' is placed 24 cm above water surface. The location of its image is at 'x' cm above the bottom of the tank. Then 'x' is



Ans : 2

$$\text{Sol : } \frac{1}{V} - \left(\frac{1}{-24} \right) = \frac{\frac{7}{4} - 1}{1} \left[\frac{1}{6} - \frac{1}{\infty} \right]$$

$$\Rightarrow V = 12$$

$$\text{apparent depth} = 12 \times \frac{4}{3} = 16$$

$$\therefore \text{distance} = 18 - 16 = 2 \text{ cm}$$

36. A series R-C combination is connected to an AC voltage of angular frequency $\omega = 500$ radian/s. If the impedance of the R-C circuit is $\sqrt{1.25}$, the time constant (in millisecond) of the circuit is

Ans : 4**Sol :** $\omega = 500 \text{ rad s}^{-1}$

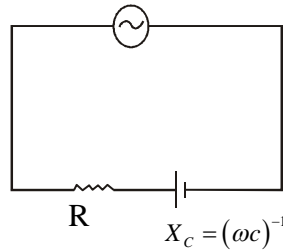
$$Z = \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}$$

$$R^2 \frac{5}{4} = R^2 + \left(\frac{1}{\omega C}\right)^2$$

$$= \frac{1}{\omega C} = \frac{R}{2}$$

$$\tau = RC = \frac{2}{\omega} = \frac{2}{500} = 0.4 \times 10^{-2}$$

$$= \boxed{4ms}$$



37. A silver sphere of radius 1 cm and work function 4.7 eV is suspended from an insulating thread in free-space. It is under continuous illumination of 200 nm wavelength light. As photoelectrons are emitted, the sphere gets charged and acquires a potential. The maximum number of photoelectrons emitted from the sphere is $A \times 10^Z$ (Where $1 < A < 10$). The value of "Z" is

Ans : 7**Sol :** $W_0 = 4.7 \text{ eV}$, $E = \frac{1240}{200} = 6.2 \text{ eV}$

$$k = 6.2 - 4.7 = 1.5 \text{ eV}$$

$$\frac{neqe}{4\pi \epsilon_0 r} + W = \frac{hc}{\lambda}$$

$$\frac{n \times 1.6 \times 10^{-19} \times 9 \times 10^9}{1 \times 10^{-2}} + 4.7 = 6.2$$

$$n \times 1.6 \times 10^{-10} \times 9 \times 10^2 = 1.5$$

$$n = \frac{1.5 \times 5}{9 \times 1.6 \times 10^{-8}} = \frac{5 \times 10^8}{48} = \frac{50}{48} \times 10^7$$

$$n = \frac{25}{24} \times 10^7$$

38. A train is moving along a straight line with a constant acceleration ' α '. A body standing in the train throws a ball forward with a speed of 10 m/s, at an angle of 60° to the horizontal. The boy has to move forward by 1.15m inside the train to catch the ball back at the initial height. The acceleration of the train, in m/s^2 , is

Ans : 5

$$\text{Sol : } T = \frac{2v_y}{a_y} = \frac{210(\sin 60)}{10} = \sqrt{3}$$

$$1.15\hat{i} = 5\sqrt{3}\hat{i} - \frac{1}{2}a(\sqrt{3})^2\hat{i}$$

$$\Rightarrow a = 5\text{ms}^{-2}$$

Section – IV(Total Marks : 16)

(Matrix-Match Type)

This section contains **2 questions**. Each question has **four statements** (A, B, C and D) given in **Column I** and **five statements** (p,q,r,s and t) in **Column II**. Any given statement in Column I can have correct matching with **ONE or MORE** statement(s) given in Column II. For example, if for a given question, statement B matches with the statements given in q and r, then for the particular question, against statement B, darken the bubbles corresponding to q and r in the ORS.

39. **Column I** shows four systems, each of the same length L , for producing standing waves. The lowest possible nature frequency of a system is called its fundamental frequency, whose wavelength is denoted as λ_f . Match each system with statements given in

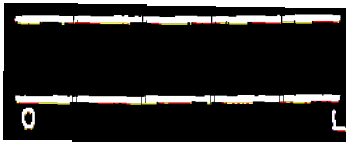
Column II describing the nature and wavelength of the standing waves.

Column I

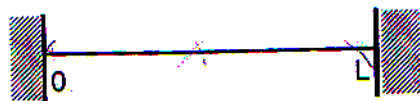
(A) Pipe closed at one end



(B) Pipe open at both ends



(C) Stretched wire clamped at both ends



(D) Stretched wire clamped at both ends and at mid-point



Column II

(p) Longitudinal waves

(q) Transverse waves

(r) $\lambda_f = L$

(s) $\lambda_f = 2L$

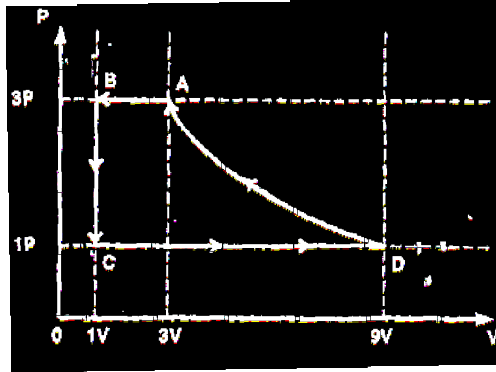
(t) $\lambda_f = 4L$

Ans : A → P, t; B → P, s; C → q, s; D → q, r

Sol : Longitudinal waves can only exist in gases while transverse waves can only exist in stretched strings

$$\therefore A \rightarrow P, t; B \rightarrow P, s; C \rightarrow q, s; D \rightarrow q, r$$

40. One mole of a monatomic ideal gas is taken through a cycle ABCDA as shown in the P.V. diagram. Column II given the characteristics involved in the cycle. Match them with each of the processes given in Column I



Column I

- (A) Process $A \rightarrow B$
 (B) Process $B \rightarrow C$
 (C) Process $C \rightarrow D$
 (D) Process $D \rightarrow A$

Column II

- (p) Internal energy decreases.
 (q) Internal energy increases.
 (r) Heat is lost
 (s) Heat is gained.
 (t) Work is done on the gas.

Ans : $A \rightarrow r, t, p$; $B \rightarrow p, r$; $C \rightarrow s, q$; $D \rightarrow t, r$

Sol : $p = k$, V, T both are decreased, so dv decreases, dw also $-ve$ so, dq is $-ve$

$V = K$, P, T dq is also $-ve$ because $dw = 0$

$P = K$, V, T increased, dv is $+ve$, dq is also $+ve$.

$T = K$ isothermal process, ' v ' is decreased, dq is $-ve$, dq is $-ve$ because, $dv = 0$

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PART III : MATHEMATICS

SECTION – I (Total Marks : 24)

(Single Correct Answer Type)

This section contains **8 multiple choice questions**. Each question has four choices (A), (B), (C) and (D) out of which **ONLY ONE** is correct

41. A value of b for which the equations

$$x^2 + bx - 1 = 0$$

$$x^2 + x + b = 0,$$

have one root in common is

a) $-\sqrt{2}$

b) $-i\sqrt{3}$

c) $i\sqrt{5}$

d) $\sqrt{2}$

Ans : B

Sol : $(c_1a_2 - c_2a_1)^2 = (a_1b_2 - a_2b_1)(b_1c_2 - b_2c_1)$

$$(-1-b)^2 = (1-b)(b^2+1)$$

$$\Rightarrow b^3 + 3b = 0 \Rightarrow b = 0, -i\sqrt{3}$$

42. Let $\omega \neq 1$ be a cube root of unity and S be the set of all non-singular matrices of the form

$$\begin{bmatrix} 1 & a & b \\ \omega & 1 & c \\ \omega^2 & \omega & 1 \end{bmatrix},$$

where each of a , b , and c is either ω or ω^2 . Then the number of distinct matrices in the set S is

a) 2

b) 6

c) 4

d) 8

Ans : A

Sol : $c - \omega c - a(\omega - c\omega^2) + b(\omega^2 - \omega^2) = 0$

$$c - (a+c)\omega + ac\omega^2 = 0 \Rightarrow (1-\omega a)(1-\omega c) \neq 0$$

$$c = 2$$

43. The circle passing through the point $(-1, 0)$ and touching the y -axis at $(0, 2)$ also passes through the point

- a) $\left(-\frac{3}{2}, 0\right)$ b) $\left(-\frac{5}{2}, 2\right)$ c) $\left(-\frac{3}{2}, \frac{5}{2}\right)$ d) $(-4, 0)$

Ans : D

Sol : $x^2 + y^2 + 2gx + 2fy + c = 0$

$$1 + 0 - 2g + c = 0$$

$$f = \pm\sqrt{c} \quad f = \pm 2$$

$$0 + 4 + 4f + c = 0$$

$$4 + c = -4\sqrt{c}$$

$$16 + c^2 + 8c = 16c$$

$$f = c^2 - 8c + 16 = 0$$

$$c = 4$$

44. If $\lim_{x \rightarrow 0} \left[1 + x \ln(1 + b^2)\right]^{\frac{1}{x}} = 2b \sin^2 \theta$, $b > 0$ and $\theta \in (-\pi, \pi]$, then the value of θ is

- a) $\pm \frac{\pi}{4}$ b) $\pm \frac{\pi}{3}$ c) $\pm \frac{\pi}{6}$ d) $\pm \frac{\pi}{2}$

Ans : D

Sol : $\lim_{x \rightarrow 0} \left(1 + x \ln(1 + b^2)\right)^{\frac{1}{x}}$

$$= e^{\lim_{x \rightarrow 0} \ln(1 + b^2)} = 1 + b^2 = 2b \sin^2 \theta$$

$$b^2 - 2b \sin^2 \theta + 1 = 0$$

$$b > 0 \quad \sin^2 \theta = 1 \Rightarrow \theta = \pm \frac{\pi}{2}$$

$$b - 1 = 0$$

$$b = 1$$

45. Let $f : [-1, 2] \rightarrow [0, \infty)$ be a continuous function such that $f(x) = f(1-x)$ for all $x \in [-1, 2]$.

Let $R_1 = \int_{-1}^2 x f(x) dx$, and R_2 be the area of the region bounded by $y = f(x)$, $x = -1$, $x = 2$, and the x-axis. Then

a) $R_1 = 2R_2$

b) $R_1 = 3R_2$

c) $2R_1 = R_2$

d) $3R_1 = R_2$

Ans : C

Sol : $R_1 = \int_{-1}^2 x f(x) dx$,

$$R_2 = \int_{-1}^2 f(x) dx$$

$$R_1 = \int_{-1}^2 f(t) dt - \int_{-1}^2 t f(t) dt$$

$$R_1 = R_2 - R_1$$

$$2R_1 = R_2$$

46. Let $f(x) = x^2$ and $g(x) = \sin x$ for all $x \in \mathbb{R}$. Then the set of all x satisfying

$(f \circ g \circ g \circ f)(x) = (g \circ g \circ f)(x)$, where $(f \circ g)(x) = f(g(x))$, is

a) $\pm\sqrt{n\pi}, n \in \{0, 1, 2, \dots\}$

b) $\pm\sqrt{n\pi}, n \in \{1, 2, \dots\}$

c) $\frac{\pi}{2} \pm 2n\pi, n \in \{\dots, -2, -1, 0, 1, 2, \dots\}$

d) $2n\pi, n \in \{\dots, -2, -1, 0, 1, 2, \dots\}$

Ans : A

Sol : $f(x) = x^2, g(x) = \sin x$

$$f(g(gf(x))) = (g(g(f(x))))$$

$$f(g(g(x^2))) = g(g(x^2))$$

$$f(g(\sin x^2)) = g(\sin x^2)$$

$$f(\sin(\sin x^2)) = \sin(\sin x^2)$$

$$\sin(\sin x^2)^2 = 1, \sin(\sin x^2) = 0$$

$$\sin(x^2) = n\pi + \frac{\pi}{2} \quad \sin x^2 = 0 = \sin 0$$

No roots

$$x^2 = n\pi$$

$$x = \sqrt{n\pi}, n \in \{0, 1, 2, \dots\}$$

47. Let (x, y) be any point on the parabola $y^2 = 4x$. Let P be the point that divides the line segment from $(0, 0)$ to (x, y) in the ratio 1:3. Then the locus of P is
- a) $x^2 = y$ b) $y^2 = 2x$ c) $y^2 = x$ d) $x^2 = 2y$

Ans : C

$$\left(\frac{at^2}{4}, \frac{2at}{4}\right) = (x, y)$$

$$\frac{t^2}{4} = x, \frac{t}{2} = y \Rightarrow t = 2y$$

$$y^2 = x \Rightarrow y^2 = x$$

48. Let P(6, 3) be a point on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$. If the normal at the point P intersects the x-axis at (9, 0), then the eccentricity of the hyperbola is

- a) $\sqrt{\frac{5}{2}}$ b) $\sqrt{\frac{3}{2}}$ c) $\sqrt{2}$ d) $\sqrt{3}$

Ans : B

$$\frac{a^2x}{x_1} + \frac{b^2y}{y_1} = a^2 + b^2$$

$$\frac{a^2x}{b} + \frac{b^2y}{3} = a^2 + b^2$$

$$\frac{3a^2}{2} = a^2 + b^2 \Rightarrow \frac{a^2}{2} = b^2$$

$$\Rightarrow e = \sqrt{\frac{3}{2}}$$

SECTION – II (Total Marks : 16)

(Multiple Correct)

This section contains **4 multiple choice questions**. Each question has four choices (A), (B), (C) and (D) out of which **ONE or MORE** may be correct

49. Let E and F be two independent events. The probability that exactly one of them occurs is $\frac{11}{25}$ and the probability of none of them occurring is $\frac{2}{25}$. If P(T) denotes the probability of occurrence of the event T, then

a) $P(E) = \frac{4}{5}, P(F) = \frac{3}{5}$

b) $P(E) = \frac{1}{5}, P(F) = \frac{2}{5}$

c) $P(E) = P(F) = \frac{2}{5}, P(F) = \frac{1}{5}$

d) $P(E) = \frac{3}{5}, P(F) = \frac{4}{5}$

Ans : AD

$$\text{Sol : } P(E^c \cap F^c) = \frac{2}{25} \Rightarrow (1-x)(1-y) = \frac{2}{25}$$

$$P(E \cap F^c) + P(E^c \cap F) = \frac{11}{25}$$

$$x(1-y) + (1-x)y = \frac{11}{25}$$

$$x + y - 2xy = \frac{11}{25}$$

$$x + y - xy = \frac{23}{25}$$

50. Let L be a normal to the parabola $y^2 = 4x$. If L passes through the point (9, 6), then L is given by

a) $y - x + 3 = 0$ b) $y + 3x - 33 = 0$ c) $y + x - 15 = 0$ d) $y - 2x + 12 = 0$

Ans : ABD

$$\text{Sol : } y = mn - 2am - am^3 \qquad y = mx - 2m - m^3$$

$$m^3 \rightarrow m + 6 = 0$$

$$m = 1$$

$$y = x - 3$$

$$y = -3x + 33$$

$$y = 2x - 12$$

51. Let $f : (0, 1) \rightarrow \mathbb{R}$ be defined by

$$f(x) = \frac{b-x}{1-bx},$$

where b is a constant such that $0 < b < 1$. Then

a) f is not invertible on (0, 1)

b) $f \neq f^{-1}$ on (0, 1) and $f'(b) = \frac{1}{f'(0)}$

c) $f = f^{-1}$ on (0, 1) and $f'(b) = \frac{1}{f'(0)}$

d) f^{-1} is differentiable on (0, 1)

Ans : CD

$$\text{Sol : } f(x) = \frac{b-x}{1-bx} = y$$

$$b-x = y - ybx$$

$$b-y = x(1-yb) \Rightarrow x = \frac{b-y}{1-by}$$

$$\Rightarrow f = f^{-1}, f^{-1}(y) = \frac{b-y}{1-by}$$

$$f'(x) = \frac{(1-bx)(-1) - (b-x)(-b)}{(1-bx)^2}$$

$$= \frac{b^2-1}{(1-bx)^2}$$

$$b \in (0, 1) \Rightarrow f'(x) < 0 \Rightarrow f \downarrow \text{ is invertible}$$

$$f'(b) = \frac{b^2-1}{(1-b^2)^2} = \frac{-1}{1-b^2} f'(0) = b^2-1$$

$$\Rightarrow f'(b) = \frac{1}{f'(0)}$$

52. If

$$f(x) = \begin{cases} -x - \frac{\pi}{2}, & x \leq -\frac{\pi}{2} \\ -\cos x, & -\frac{\pi}{2} < x \leq 0 \\ x-1, & 0 < x \leq 1 \\ \ln x, & x > 1, \end{cases}$$

then

a) $f(x)$ is continuous at $x = -\frac{\pi}{2}$

b) $f(x)$ is not differentiable at $x = 0$

c) $f(x)$ is differentiable at $x = 1$

d) $f(x)$ is differentiable at $x = -\frac{3}{2}$

Ans : ABCD

$$\text{Sol : } \lim_{x \rightarrow \frac{-\pi}{2}} f(x) = 0$$

$$\lim_{x \rightarrow \frac{-\pi}{2}} f(x) = 0$$

$$\lim_{x \rightarrow \frac{-\pi}{2}^+} f(x) = \lim_{x \rightarrow \frac{-\pi}{2}^-} f(x) = 0$$

$$\Rightarrow \text{'f' is continuous at } x = \frac{-\pi}{2}$$

$$f'(x) = \begin{cases} -1 & x \leq -\frac{\pi}{2} \\ \sin x & -\frac{\pi}{2} < x < 0 \\ 1 & 0 < x \leq 1 \\ \frac{1}{x} & x > 1 \end{cases}$$

$$\Rightarrow f \text{ is not diff at } x = 0 \quad \because Lf'(0) = 0, Rf'(0) = 1$$

$$Lf'(1) = Rf'(1) \Rightarrow f \text{ is diff at } x = 1$$

$$f \text{ is diff at } x = -\frac{3}{2}$$

SECTION – III**(INTEGER ANSWER TYPE)**

This section contains **6 questions**. The answer to each of the questions is a **single-digit integer**, ranging from 0 to 9. The bubble corresponding to the correct answer is to be darkened in the ORS.

53. The number of distinct real roots of $x^4 - 4x^3 + 12x^2 + x - 1 = 0$ is

Ans: 2

$$\text{Sol : } f(x) = x^4 - 4x^3 + 12x^2 + x - 1 = 0$$

$$f'(x) = 4x^3 - 12x^2 + 24x + 1$$

If it is strictly increasing

$$\therefore f'(x) = 0 \text{ has only one root}$$

$$f(0) < 0$$

$$f(1) > 0$$

54. Let $y'(x) + y(x)g'(x)$, $y(0) = 0$, $x \in \mathbb{R}$, where $f'(x)$ denotes $\frac{df(x)}{dx}$ and $g(x)$ is a given non-constant differentiable function on \mathbb{R} with $g(0) = g(2) = 0$. Then the value of $y(2)$ is

Ans : 0

Sol : $y'(x) + y(x)g'(x) = g(x)g'(x)$

$$ye^{\int g'(x)dx} = \int g(x)g'(x)e^{\int g'(x)dx}$$

$$ye^{g(x)} = \int g(x)g'(x)e^{g(x)} dx$$

$$= \int g(x)e^{g(x)}g'(x) dx$$

$$g(x) = t$$

$$g'(x)dx = dt$$

$$\int te^t dt$$

$$ye^{g(x)} = g(x)e^{g(x)} - e^{g(x)} + c$$

$$y(0) = 0 - 1 + c \Rightarrow c = 1$$

$$y(2)e^{g(2)} = g(2)e^{g(2)} - e^{g(2)} + 1$$

$$y(2) = 0$$

55. Let M be a 3×3 matrix satisfying

$$M \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}, \quad M \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}, \quad \text{and} \quad M \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 12 \end{bmatrix}$$

Then the sum of the diagonal entries of M is

Ans : 9

Sol : $M \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix} \quad M \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 12 \end{bmatrix}$

$$M \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$$

$$M = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$$

$$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}$$

$$\Rightarrow b = -1$$

$$e = 2$$

$$h = 3$$

$$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$$

$$a - b = 1 \Rightarrow a = 0$$

$$g - h = -1 \Rightarrow g = 2$$

$$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 12 \end{bmatrix}$$

$$\Rightarrow g + h + i = 12$$

$$\Rightarrow i = 12 - 2 - 3 = 7$$

$$a + e + i = 0 + 2 + 7 = 9$$

56. Let $\vec{a} = -\hat{i} - \hat{k}$, $\vec{b} = -\hat{i} + \hat{j}$ and $\vec{c} = \hat{i} + 2\hat{j} + 3\hat{k}$ be three given vectors. If \vec{r} is a vector such that $\vec{r} \times \vec{b} = \vec{c} \times \vec{b}$ and $\vec{r} \cdot \vec{a} = 0$, then the value of $\vec{r} \cdot \vec{b}$ is

Ans : 9

Sol : $(\vec{r} - \vec{c}) \times \vec{b} = \vec{0}$

$$\vec{r} - \vec{c} \parallel \vec{b}$$

$$\vec{r} = \vec{c} + t\vec{b}$$

$$\vec{r} \cdot \vec{a} = 0 \Rightarrow \vec{c} \cdot \vec{a} + t\vec{b} \cdot \vec{a}$$

$$t = -\frac{\vec{c} \cdot \vec{a}}{\vec{b} \cdot \vec{a}}$$

$$\vec{r} = \vec{c} - \frac{\vec{a} \cdot \vec{c}}{\vec{a} \cdot \vec{b}} \vec{b}$$

$$= (i + 2j + 3k) - \left(\frac{-4}{1}\right)(-i + j)$$

$$i + 2j + 3k - 4i + 4j$$

$$= -3i + 6j + 3k$$

$$\vec{r} \cdot \vec{b} = 3 + 6 = 9$$

57. The straight line $2x - 3y = 1$ divides the circular region $x^2 + y^2 \leq 6$ into two parts. If

$S = \left\{ \left(2, \frac{3}{4}\right), \left(\frac{5}{2}, \frac{3}{4}\right), \left(\frac{1}{4}, -\frac{1}{4}\right), \left(\frac{1}{8}, \frac{1}{4}\right) \right\}$, then the number of points in S lying inside the smaller part is

Ans : 2

Sol : (0,0) lies at side smaller part

$$-0 - 1 < 0$$

$$2x - 3y - 1$$

$$\left(2, \frac{3}{4}\right) \quad 4 - \frac{9}{4} - 1 > 0$$

$$\left(\frac{5}{2}, \frac{3}{4}\right) \quad 5 - \frac{9}{4} - 1 > 0$$

$$\left(\frac{1}{4}, -\frac{1}{4}\right) \quad \frac{5}{4} - 1 > 0$$

$$\left(\frac{1}{8}, \frac{1}{4}\right) \quad \frac{1}{4} - \frac{3}{4} < 0$$

Number of points = 2

58. Let $\omega = e^{\frac{i\pi}{3}}$, and a, b, c, x, y, z be non zero complex numbers such that $a + b + c = x$
 $a + b\omega + c\omega^2 = y$ then $a + b\omega^2 + c\omega = z$
 the value of $\frac{|x|^2 + |y|^2 + |z|^2}{|a|^2 + |b|^2 + |c|^2}$ is

Ans : Doesn't get the integer

59. Match the statements given in Column I with the interval/union of interval given in column II

Column I

Column II

A) The set $\left\{ \operatorname{Re} \left(\frac{2iz}{1-z^2} \right) : Z \text{ is a complex number, } |z|=1, z \neq \pm 1 \right\}$ p) $(-\infty, -1) \cup (1, \infty)$

B) The domain of the function $f(x) = \sin^{-1} \left(\frac{8(3)^{x-2}}{-3^{2(x-1)}} \right)$ is q) $(-\infty, 0) \cup (0, \infty)$

C) If $f(\theta) = \begin{vmatrix} 1 & \tan \theta & 1 \\ -\tan \theta & 1 & \tan \theta \\ -1 & -\tan \theta & 1 \end{vmatrix}$, then the set $\left\{ f(\theta) : 0 \leq \theta < \frac{\pi}{2} \right\}$ is r) $[2, \infty)$

D) If $f(x) = x^{\frac{3}{2}}(3x-10), x \geq 0$, then $f(x)$ is increasing in s) $(-\infty, -1] \cup [1, \infty)$

t) $(-\infty, 0] \cup [2, \infty)$

Sol : A) $\frac{2iz}{1-z^2} = \frac{2ie^{i\theta}}{1-e^{2i\theta}}$
 $= \frac{2ie^{i\theta}}{1 - \{\cos 2\theta + i \sin 2\theta\}}$
 $= \frac{2i(\cos \theta + i \sin \theta)}{2 \sin^2 \theta - 2i \sin \theta \cos \theta}$
 $= \frac{2i(\cos \theta + i \sin \theta)}{2 \sin \theta (\sin \theta - i \cos \theta)}$

$$= \frac{-1}{\sin \theta}$$

$$-1 \leq \sin \theta \leq 1$$

$$\Rightarrow \frac{-1}{\sin \theta} \in (-\infty, -1] \cup [1, \infty)$$

Ans : A - s ;

B) Let $3^x = a$

$$\sin^{-1}\left(\frac{8 \cdot 3^{x-2}}{1 - 3^{2(x-1)}}\right) \text{ defined for}$$

$$-1 \leq \frac{8 \cdot 3^{x-2}}{1 - 3^{2(x-1)}} \leq 1$$

$$\text{let } 3^x = a \Rightarrow -1 \leq \frac{8a}{9 - a^2} \leq 1$$

$$\Rightarrow x \in (-\infty, 0] \cup [2, \infty)$$

B - t ;

$$\text{C) } f(\theta) = (1 + \tan^2 \theta) - \tan \theta(0) + 1(\tan^2 \theta + 1)$$

$$= 2 \sec^2 \theta$$

$$0 \leq \theta < \frac{\pi}{2} \Rightarrow 2 \sec^2 \theta \in [2, \infty)$$

C - r ;

$$\text{D) } f'(x) = x^{\frac{3}{2}} \cdot 3 + (3x - 10) \cdot \frac{3}{2} x^{\frac{1}{2}}$$

$$= 3\sqrt{x} \frac{(2x + 3x - 10)}{2} \geq 0$$

$$3\sqrt{x} \frac{(5x - 10)}{2} \geq 0$$

$$x > 0, x \geq 2$$

D - r

60. Match the statements given in Column I with the values given in Column II

Column IColumn II

A) If $\vec{a} = \hat{j} + \sqrt{3}\hat{k}$, $\vec{b} = -\hat{j} + \sqrt{3}\hat{k}$ and $\vec{c} = 2\sqrt{3}\hat{k}$

p) $\frac{\pi}{6}$

form a triangle, then the internal angle of the triangle between \vec{a} and \vec{b} is

B) If $\int_a^b (f(x) - 3x) dx = a^2 - b^2$ then the value of $f\left(\frac{\pi}{6}\right)$ is

q) $\frac{2\pi}{3}$

C) The value of $\frac{\pi^2}{\ln 3} \int_{\frac{7}{6}}^{\frac{5}{6}} \sec(\pi x) dx$ is

r) $\frac{\pi}{3}$

D) The maximum value of $\left| \text{Arg} \left(\frac{1}{1-z} \right) \right|$ for $|z|=1$, $z \neq 1$ is

s) π

given by

t) $\frac{\pi}{2}$

A) $\cos \theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| |\vec{b}|} = \frac{-1+3}{2.2} = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3}$

As A - q ;

B) (1) $f(b) - 3b = -2b$

$f(b) = b \Rightarrow f\left(\frac{\pi}{6}\right) = \frac{\pi}{6}$

B - p ;

C) $\frac{\pi^2}{\ln 3} \ln \left(\left| \frac{\sec \pi x + \tan \pi x}{\pi} \right| \right)_{\frac{7}{6}}^{\frac{5}{6}}$

$\frac{\pi}{\ln 3} (\ln \sqrt{3}) = \frac{\pi}{2}$

C - s

$$D) |\arg(1) - \arg(1-z)|$$

$$|0 - \arg((1 - \cos \theta) - i \sin \theta)|$$

$$\left| -\tan^{-1}\left(\frac{-\sin \theta}{1 - \cos \theta}\right) \right| = \left| \tan^{-1}\left(\cot \frac{\theta}{2}\right) \right| < \frac{\pi}{2}$$

∴ As D-t ;

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